

### Competition 2.4: Exploring scaling analysis

**Part 1:** Consider two volcanic explosions. Assume the air density is the same, and the blast radius of the first explosion is twice that of the second explosion in the same time period. How many times more energy would you expect to be released in the first explosion compared with the second?

**Part 2:** We can use scaling analysis in all sorts of situations. Suppose there is a volcanic eruption on an island, and you need to leave quickly by rowing boat. It turns out we can use scaling analysis to see how the velocity  $v$  of the boat depends on the number of rowers  $N$ .

The drag caused by the friction as the boat travels through the water is a force  $F$  proportional to  $A v^2$  where  $A$  is the submerged cross-sectional area of the boat:

$$F \sim A v^2$$

This means we'll need a total power  $P$  to overcome this drag:

$$P = F v \sim A v^3$$

Assuming all the rowers weigh the same, Archimedes' Law tells us that the displaced volume  $V$  is proportional to  $N$  (we assume that the mass of the boat is negligible compared to the rowers). Therefore:

$$A \sim V^{2/3} \sim N^{2/3}$$

Assuming also that all rowers produce the same power, we have:

$$P \sim N$$

Putting this all together, how does  $v$  scale with  $N$ ?

(This analysis was first performed by T. McMahon in 1971, and it agrees pretty well with Olympic rowing times.)

### **How to enter**

1. Complete your competition entry and make sure it follows the guidelines listed below
2. Fill out the [Competition Cover Sheet](#)
3. Email your entry and your cover sheet to us at [inspire@sjc.ox.ac.uk](mailto:inspire@sjc.ox.ac.uk)

### **Terms & Conditions**

1. You must complete the [Competition Cover Sheet](#) and submit it along with your entry to [inspire@sjc.ox.ac.uk](mailto:inspire@sjc.ox.ac.uk) before the closing date. If we do not receive a completed cover sheet with your entry, we will NOT be able to consider the entry.
2. The competition closes at **5pm on Monday 24 February 2020**. Once you submit your entry you will receive an automated confirmation that your entry has been submitted.
3. The work you submit must be entirely your own and should show the work you did to arrive at your answer. We will accept a typed version of your work, or a scan or photo of handwritten work as long as it is clearly legible.
4. Competition winners will be contacted via email about a week after the competition's closing date. The following prizes will be awarded:
  - a. The first 5 pupils to submit the correct answers will receive a £10 voucher each
  - b. Everyone who submits the correct answers will have their name listed on Inspire Digital